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$$\frac{[rb + (n-r+1)a][(n-r+1)b + 2a]}{ab(n+1)^2}, \dots, \text{the series to be summed.}$$

$$\text{Hence } S = \frac{ab(n^2+1) + n(a^2+b^2)}{ab(n+1)^2} + \frac{ab[(n-1)^2+r^2] + 2(n-1)(a^2+b^2)}{ab(n+1)^2} + \dots$$

$$+ \frac{ab[(n-r+1)^2+r^2] + r(n-r+1)(a^2+b^2)}{ab(n+1)^2} + \dots$$

$$= \frac{1}{ab(n+1)^2} \{ ab[n^2 + (n-1)^2 + \dots + 1^2] + ab[1^2 + 2^2 + \dots + n^2] \}$$

$$= \frac{(a^2+b^2)[n+2(n-1)+3(n-2)+\dots+r(n-r+1)+\dots+n]}{ab(n+1)^2}$$

$$= \frac{1}{ab(n+1)^2} \left[2ab \frac{n(n+1)(2n+1)}{6} + (a^2+b^2) \left(\frac{n(n^2+1)}{2} - \frac{n(n^2-1)}{3} \right) \right]$$

$$= \frac{1}{ab(n+1)^2} \left[2ab \frac{n(n+1)(2n+1)}{6} + (a^2+b^2) \left(\frac{n(n+1)(n+2)}{6} \right) \right]$$

$$= \frac{n}{ab(n+1)^2} \left[\frac{6ab(n+1)^2}{6} - \frac{n^2+3n+2}{6} \cdot 2ab + \frac{(n+1)(n+2)}{6} (a^2+b^2) \right]$$

$$= n \left[1 + \frac{a^2-2ab+b^2}{6ab(n+1)^2} \cdot (n+1)(n+2) \right] = n \left[1 + \frac{n+2}{n+1} \cdot \frac{(a-b)^2}{6ab} \right].$$

Solved similarly by G. B. M. Zerr, and J. Scheffer.

GEOMETRY.

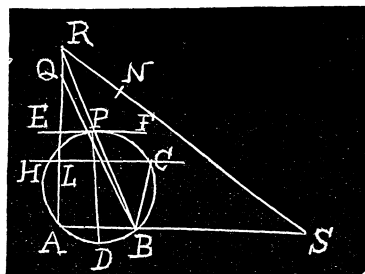
352. Proposed by G. I. HOPKINS, Professor of Astronomy, High School, Manchester, N. H.

Required, to construct the triangle, having given the base, vertical angle and sum of the altitude and the two remaining sides.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let $AB=a$ be the given base; ACB the given vertical angle; p —to the sum of the altitude and the remaining sides. On AB describe the seg—

ment APB containing the given angle. Draw the diameter PD perpendicular to AB . Also draw AQ perpendicular to AB . Draw BP meeting AQ in Q . With B as a center, and a radius equal to p , describe an arc cutting AQ produced in R . With R as a center, and a radius equal to $p + AQ$, describe an arc cutting AB produced in S . On SR measure off $SN = SA$. Then RN



= the altitude. Take $AL = RN$, and draw HLC parallel to AB , cutting the circle in C . Draw AC, BC . Then ACB is the required triangle. For let x, y, z be the sides BC, AC , and the altitude. Then $xy \sin C = az$, $x + y + z = p$, $a^2 = x^2 + y^2 - 2xy \cos C$.

$$\therefore a^2 + 2xy(1 + \cos A) = (p - z)^2. \quad \therefore z^2 - 2(p + a \cot \frac{1}{2} C) = a^2 - p^2.$$

$$\therefore z = p + a \cot \frac{1}{2} C - \sqrt{[(p + a \cot \frac{1}{2} C)^2 - (p^2 - a^2)]}.$$

$$\angle AQB = \frac{1}{2} C, \quad AQ = a \cot \frac{1}{2} C, \quad RS = p + a \cot \frac{1}{2} C, \quad AR = \sqrt{(p^2 - a^2)}, \quad AS = \sqrt{[(p + a \cot \frac{1}{2} C)^2 - (p^2 - a^2)]}.$$

$$\therefore RN = z. \quad \therefore \text{The triangle } ACB \text{ contains all the required parts.}$$

Also solved by J. Scheffer.

353. Proposed by L. H. McDONALD, M. A., Ph. D., Sometimes Tutor at Cambridge, Jersey City, N. J.

In a given circle place two chords which shall be in a given ratio and also a given distance apart.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let the ratio $m:n$ to distance apart $= d$; the radius of the given circle $= r$; u, v the distances of the chords from the center.

Then $2\sqrt{(r^2 - u^2)}$, $2\sqrt{(r^2 - v^2)}$ are the lengths of the chords.

$$\therefore m\sqrt{(r^2 - v^2)} = n\sqrt{(r^2 - u^2)}, \text{ or } (m^2 - n^2)r^2 = m^2v^2 - n^2u^2, \text{ and } u + v = d.$$

$$\therefore u = \frac{m^2 d - \sqrt{[r^2(m^2 - n^2)^2 + m^2 n^2 d^2]}}{m^2 - n^2},$$

$$v = \frac{\sqrt{[r^2(m^2 - n^2)^2 + m^2 n^2 d^2]} - n^2 d}{m^2 - n^2}.$$

Hence, if $AB = d$, take $AC = u$, $CB = v$, and with C as center and radius r describe a circle, through A and B perpendicular to AB draw lines intersecting this circle. The chords of the circle formed by these lines are the chords required.

Also solved by S. A. Corey.

354. Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, England.

Find the condition that triangles which are circumscribed to one of two confocal parabolas may be inscribed in the other.